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VARIABILITY IN FLIGHT-BY-FLIGHT  
CRACK GROWTH RATE BEHAVIOR

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## FOREWARD

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## ABSTRACT

The initial results of a variable amplitude fatigue crack growth rate test program are presented. The test program was designed to study the influence that the stress intensity factor gradient ( $dK/da$ ) had on crack growth rate behavior in 7075-T6 aluminum alloy. Constant stress intensity factor conditions were used to study this behavior, using a twenty-four inch wide center-cracked panel.

Based on earlier work by Hillberry on constant amplitude crack growth rate behavior, a three-parameter lognormal distribution function was selected to describe the crack growth rate data. Comparison of crack lengths showed no gradient effect. Parameters of the statistical distribution were compared using the secant and incremental polynomial methods of differentiation and for different flight intervals achieved by removing points prior to the differentiation process.

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## SECTION I

### INTRODUCTION

Flight-by-flight crack growth rate ( $\Delta a/\Delta F$ ) data are important to the structural design of fatigue critical aerospace components. This paper documents an investigation of the variability associated with flight-by-flight crack growth rate behavior. Variability in crack growth rate data occurs because of error in crack length measurement, lack of precise load and environmental control, as well as variability in material behavior. The type of differentiation technique employed also effects the reported variability. Measurement, load, and environmental discrepancies can be minimized by standardizing test procedures. For example, choice of the proper crack length measurement interval will help to minimize the relative size of the measurement error and thereby decrease the reported variability. The influence of variations in test parameters should be realized when employing crack growth rate test data for design purposes, as reported by Hillberry<sup>(1)</sup>, Wei<sup>(2)</sup> and others.

In order to establish baseline crack growth rate data for this study, a variable amplitude load history was applied to a twenty-four inch center-cracked panel. From earlier experience, this particular loading sequence was known to produce steady state crack growth rate behavior<sup>(3,4)</sup>. Steady state behavior is similar to that experienced under constant amplitude fatigue loading.

A "short" load history was chosen so that the influence of the stress intensity factor gradient ( $dK/da$ ) on the crack growth rate behavior



in 7075-T6 aluminum alloy could be studied. Load shedding techniques were employed to obtain constant stress intensity factor conditions to study the resulting variability. A "short" load history has been defined by Elber<sup>(5)</sup> as one in which the crack growth during one repeat load interval is less than the plastic zone size.

In this investigation, flight-by-flight crack growth rate data resulting from the secant method of differentiation were fit to four distribution functions. This was done to select the function which best describes the distribution of the  $\Delta a/\Delta F$  data. A measure of the variability was then obtained in terms of the distribution parameters. It was first necessary to show that the variability did not change as the crack advanced under constant stress intensity factor conditions. Increasing the crack length measurement interval substantially decreased the variability.

## SECTION 2

### TEST DESCRIPTION

#### 2.1 Load History

A modified design stress history for bomber type aircraft, containing 135,000 cycles per lifetime, was used for this test program. The original stress history was composed of three mission blocks as studied by Potter<sup>(6)</sup>. The first contained only once per flight load events. The second and third contained all load events of the first plus additional load events which occurred once in ten flights and once in one hundred flights, respectively. For the purpose of establishing steady-state crack growth rate behavior, the higher periodic loads were omitted; only the once per flight load events were included in the initial repetitive sequence. All compression loads were taken out of the load history (see Figure 1).

The load history was such that the crack never grew out of the plastic zone in a single flight. The highest load appearing in the modified load history was 88 percent of the highest load level which occurred in the original history. Because of the unique relation between loads in the load history, constant stress intensity factor conditions were accomplished through the reduction of the maximum repeating load as suggested by the Feddersen secant formula<sup>(7)</sup>. This insured that the maximum stress intensity factor,  $K_{max}$ , was kept constant within one percent as shown in Figure 2.

#### 2.2 Material, Geometry and Test Methods

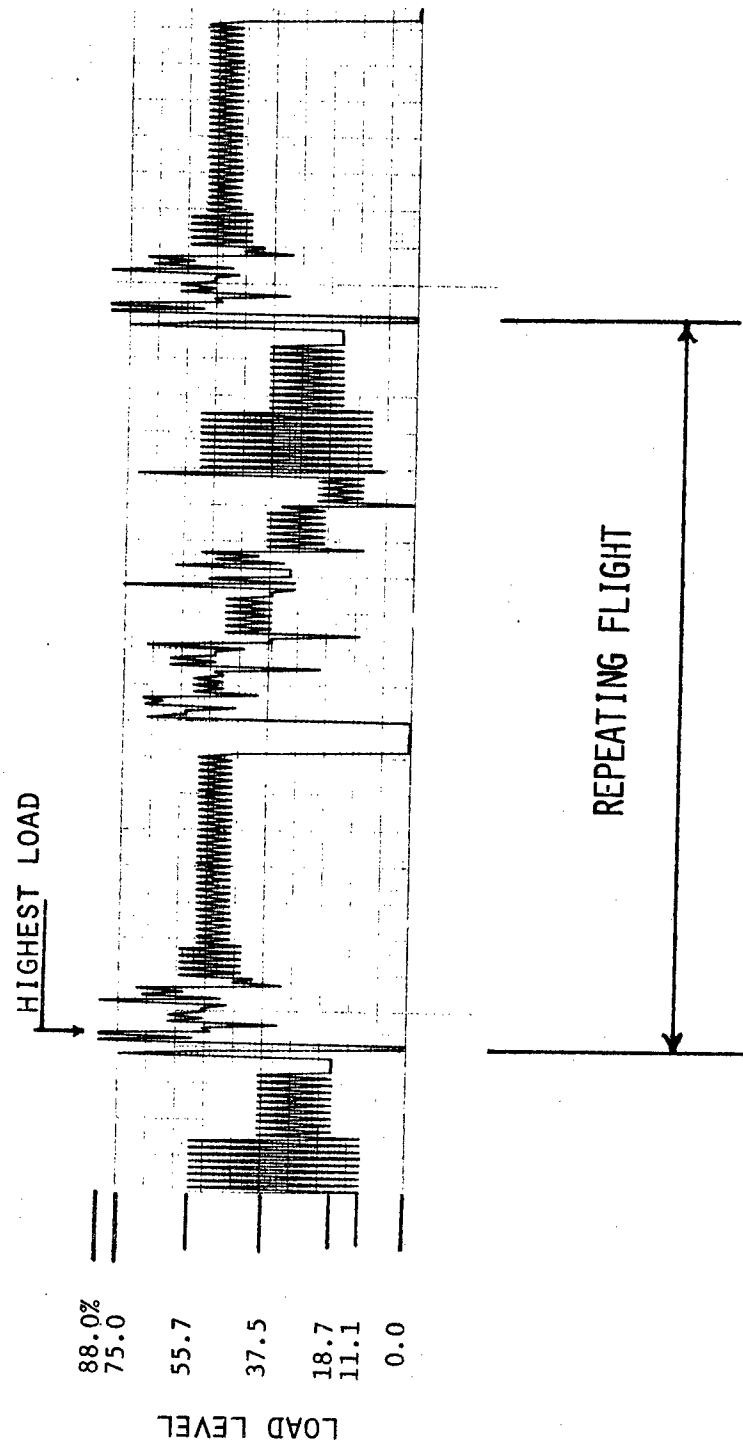


FIGURE 1: LOAD HISTORY

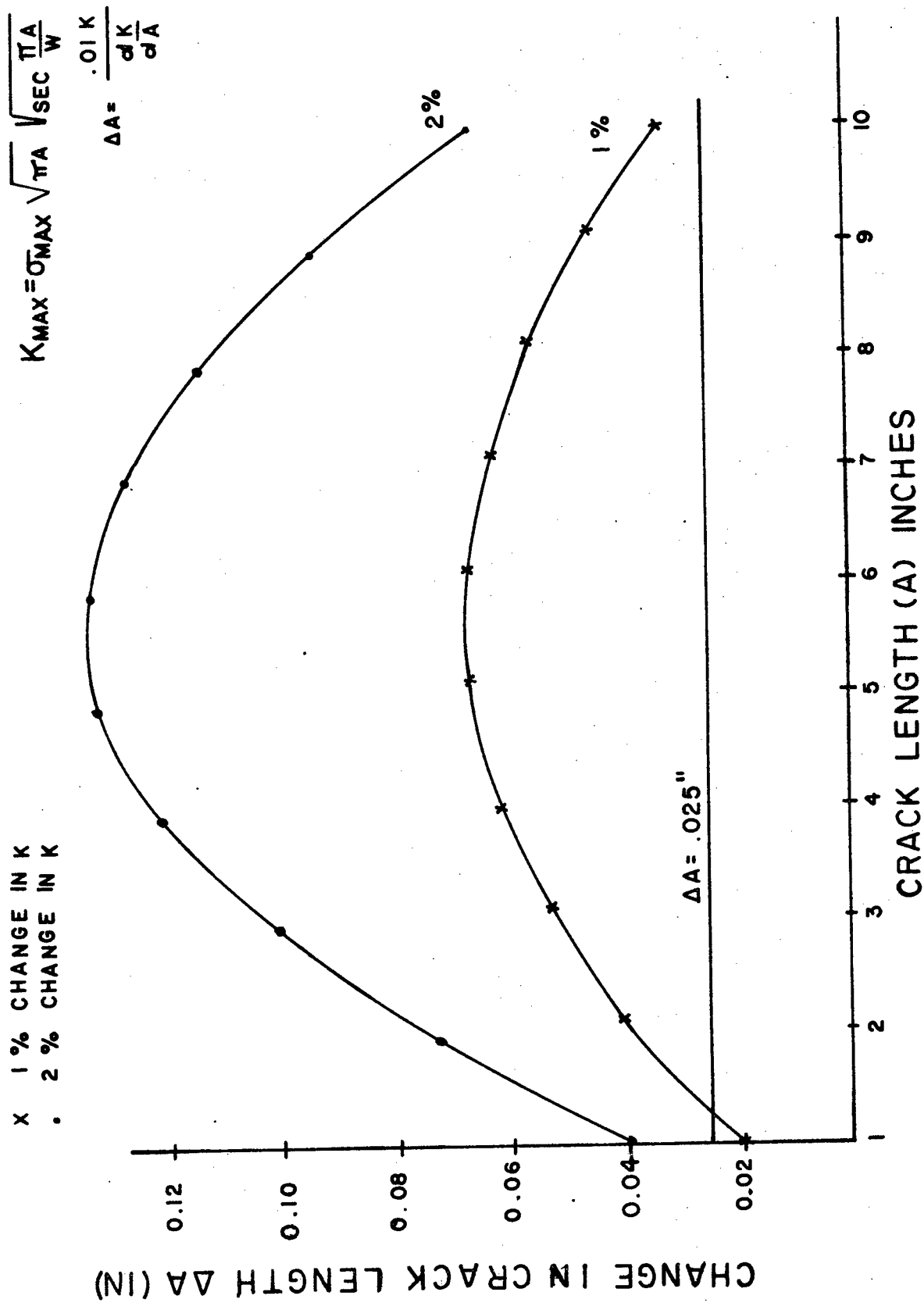


FIGURE 2. RELATIONSHIP BETWEEN CHANGE IN CRACK LENGTH AND CRACK LENGTH

### 2.2.1 Material and Specimen Geometry

A 24 in. (610 mm) wide center-cracked panel, manufactured from 0.182 in. (4.6 mm) thick 7075-T6 aluminum alloy was used (see Figure 3). The central notch was 1.663 in. (42 mm) in length.

### 2.2.2 Stress Intensity Factor Coefficient

Because the stress intensity factor is directly related to the crack growth rate  $\Delta a/\Delta F$ , the flight-by-flight crack growth rate behavior can be predicted for any level of the maximum stress intensity factor  $(K_{\max})^{(4)}$ . This functional relationship can be described by:  $\frac{da}{dF} = C(K_{\max})^{(n)}$  which is applicable to steady state spectra<sup>(3)</sup>. A maximum stress intensity factor  $(K_{\max}^P)$  of 30 ksi  $\sqrt{\text{in}}$  (33 MP<sub>a</sub>  $\sqrt{\text{m}}$ ) was chosen to correspond with the maximum applied load level in the spectrum and was controlled within one percent through load shedding. By keeping the stress intensity factor "quasi" constant, a determination can be made of which distribution function  $\Delta a/\Delta F$  follows at a particular stress intensity factor level.

### 2.2.3 Load Shedding

The maximum stress intensity factor was kept "constant" within one percent by reducing the load levels for every 0.050 in. (1.27 mm) of total (2a) crack length. This was determined as follows: to keep  $K_{a+\Delta a}$  within one percent of  $K_a$ , a first order approximation was used.

$$K_{a+\Delta a} \leq 1.01 K_a$$

$$K_a + \frac{dK}{da} [(a+\Delta a)-a] \leq 1.01 K_a$$

$$\Delta a \leq 0.01 K_a \frac{da}{dK}$$

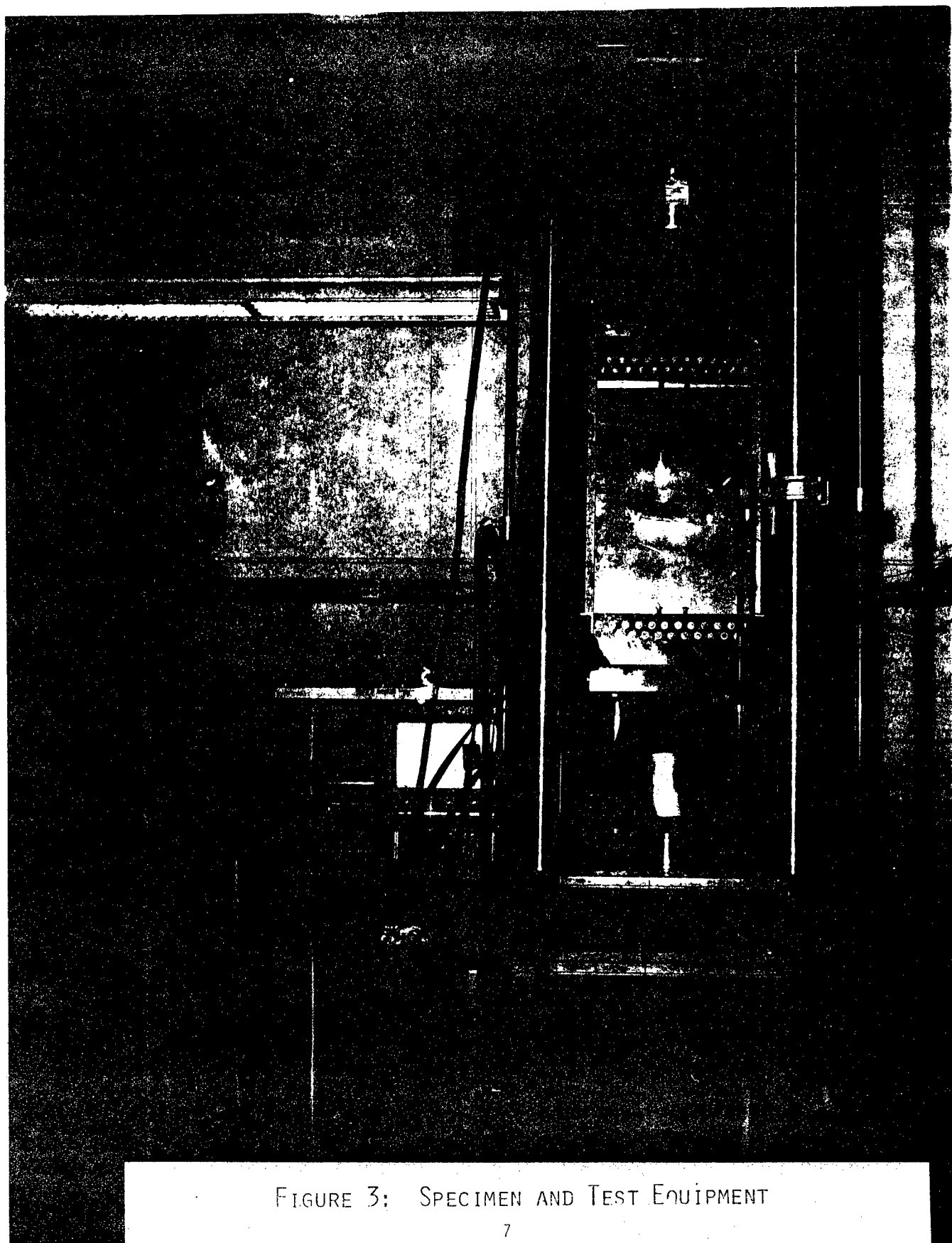


FIGURE 3: SPECIMEN AND TEST EQUIPMENT

For a center-cracked panel,

$$K_{\max} = \frac{P_{\max}}{B} \frac{\sqrt{\pi a}}{W} \sqrt{\frac{\sec \pi a}{W}}$$

$$\Delta a = 0.02a \left[ \frac{W}{W + \pi a \tan \frac{\pi a}{W}} \right]$$

where  $P$  = load;  $B$  = thickness of plate; and  $W$  = width of plate. A value of 0.025 in (.635 mm) for  $\Delta a$  was selected as the growth increment for load shedding (Figure 2). For most of the data collected, the change in  $K_{\max}$  was less than one percent.

#### 2.2.4 Test Equipment

A 500 kip (2.2MN) static, 250 kip (1.1MN) dynamic capacity load frame was used to subject test specimens to flight-by-flight loading previously described. The load frame employed the principles of closed-loop servocontrol. The load levels and cycle shape were stored in a 4096 byte memory digital programmer and then fed to the servo- load controllers. The 500 kip load cell connected in series with the test specimen provided a feedback signal for load control.

The applied test loads were monitored through an independent data system and were maintained to within one percent of the programmed value. The crack length was measured using a binocular zoom microscope with a maximum magnification of 40X. A Mylar scale calibrated in 0.005 in (0.127 mm) divisions was attached to the specimen. Crack length readings were estimated to be within  $\pm 0.002$  inches (.0508 mm). This value will be referred to as the measurement precision,  $\epsilon$ .

### SECTION 3

#### DISCUSSION AND RESULTS

In the initial test, the crack length was measured after every fifteen flights. The crack length versus flight data was tabulated and put into a statistical analysis program provided by Purdue University under an AFOSR grant<sup>(8)</sup>. From the statistical analysis of  $\Delta a/\Delta F$  a determination was made as to which of four (4) statistical distributions best described the data. The data are shown in Figure 4.

Crack growth rates were generated using the secant method of differentiation. The differentiated data are shown in Figure 5. The resulting  $\Delta a/\Delta F$  data were fit to four distributions: normal, 2 parameter lognormal, 3 parameter lognormal and 3 parameter Weibull. Of the four distributions, the 3 parameter lognormal was found to have the highest correlation coefficient for all the data sets (Table I and Figure 6). Because of the high correlation coefficients associated with the four distributions, all appear adequate for comparing the variability associated with one set of crack growth rate data.

The  $\Delta a/\Delta F$  data from the first panel were grouped into 0.5 inch intervals (Figure 7). The sample means of the groups remained fairly constant as the crack progressed. The means were indistinguishable from one another using the student's t test at a 95% confidence level. These four groups consist of all the data collected in the first panel. Each group contains between forty and sixty  $\Delta a/\Delta F$  observations. Because the sample means are indistinguishable as the crack advanced, a distribution can be found which describes the variation  $\Delta a/\Delta F$  data for all crack lengths measured for a given number of flights.



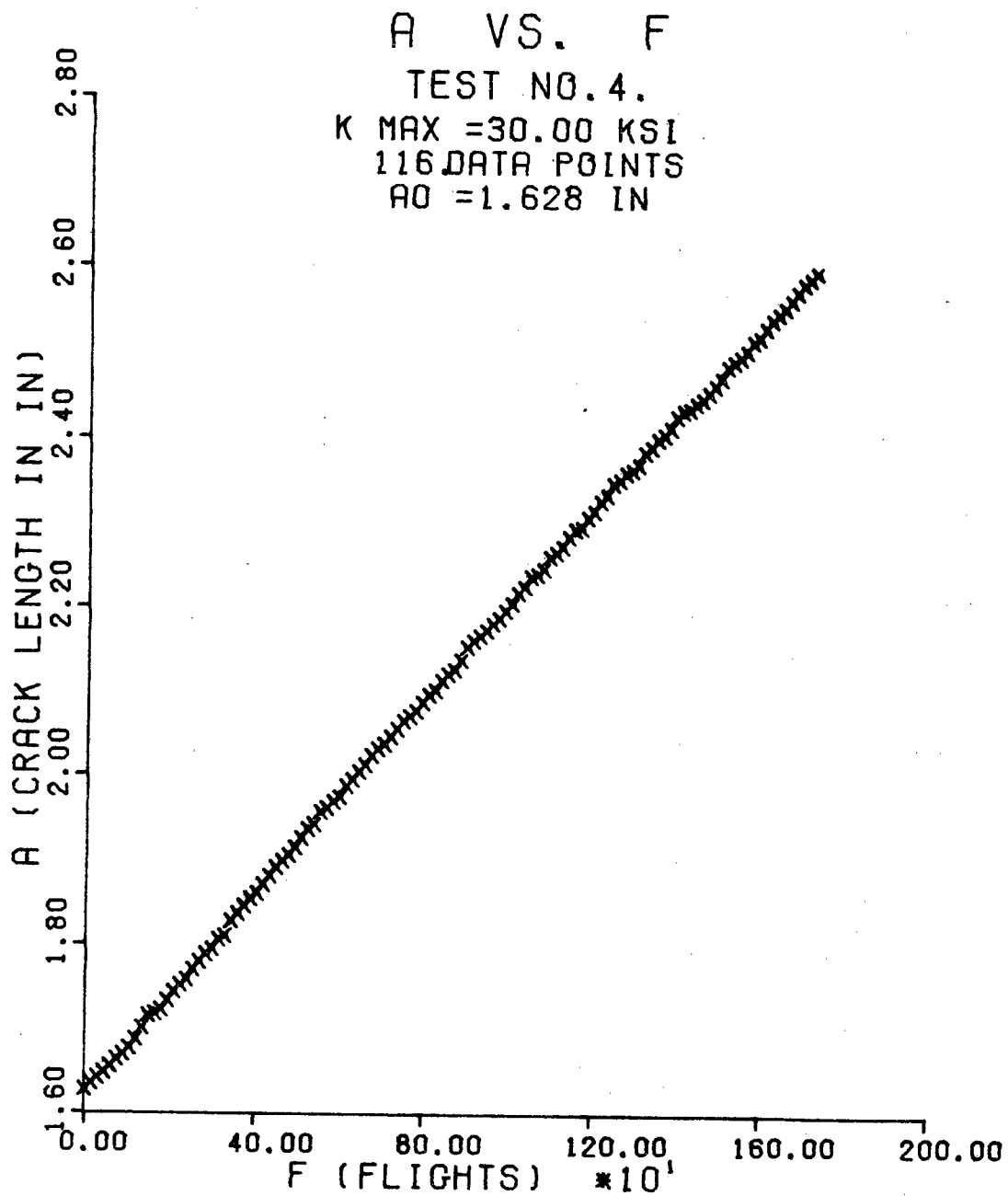


FIGURE 4: CONSTANT STRESS INTENSITY FACTOR CONTROL CONDITIONS, CRACK LENGTH VS. REPEATING FLIGHTS

TABLE I Correlation Coefficients - for Two Methods of Differentiation

DISTRIBUTION FUNCTIONS

Secant Method

Effective Measurement Frequency ( $\Delta F$ ) Flights	No. of Pts Considered	Normal	2-Lognormal	3-Lognormal	Weibull
15	116	.9912	.9981	.9998	.9936
30	58	.9859	.9992	.9998	.9936
45	38	.9949	.9999	1.0000	.9976

Incremental Polynomial Method

15	116	.9960	.9999	1.0000	.9987
30	58	.9767	.9999	1.0000	.9991

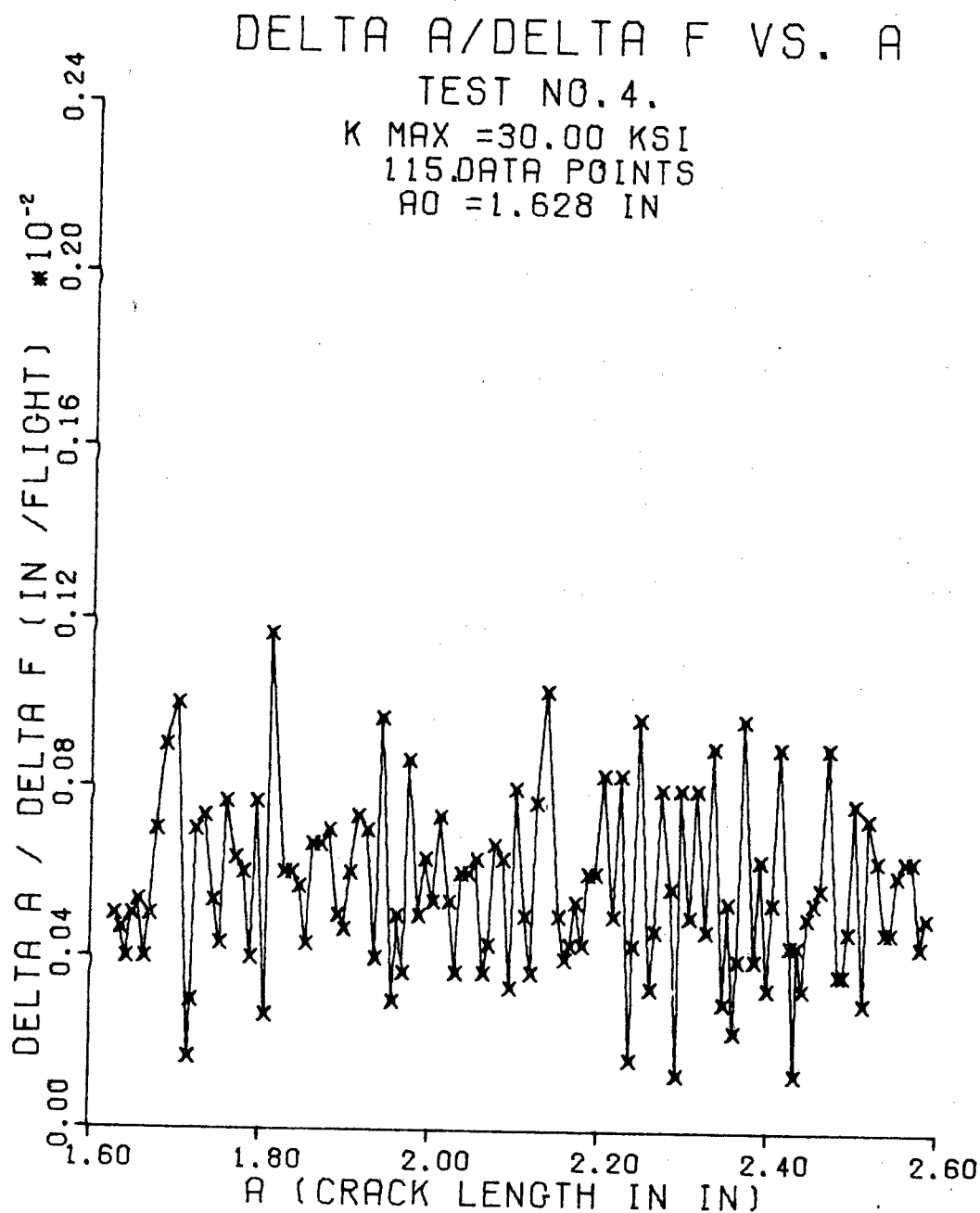


FIGURE 5: CONSTANT  $K_{MAX}$  CONTROL CONDITIONS, CRACK GROWTH RATE DATA AS A FUNCTION OF CRACK LENGTH USING SECANT METHOD OF DIFFERENTIATION

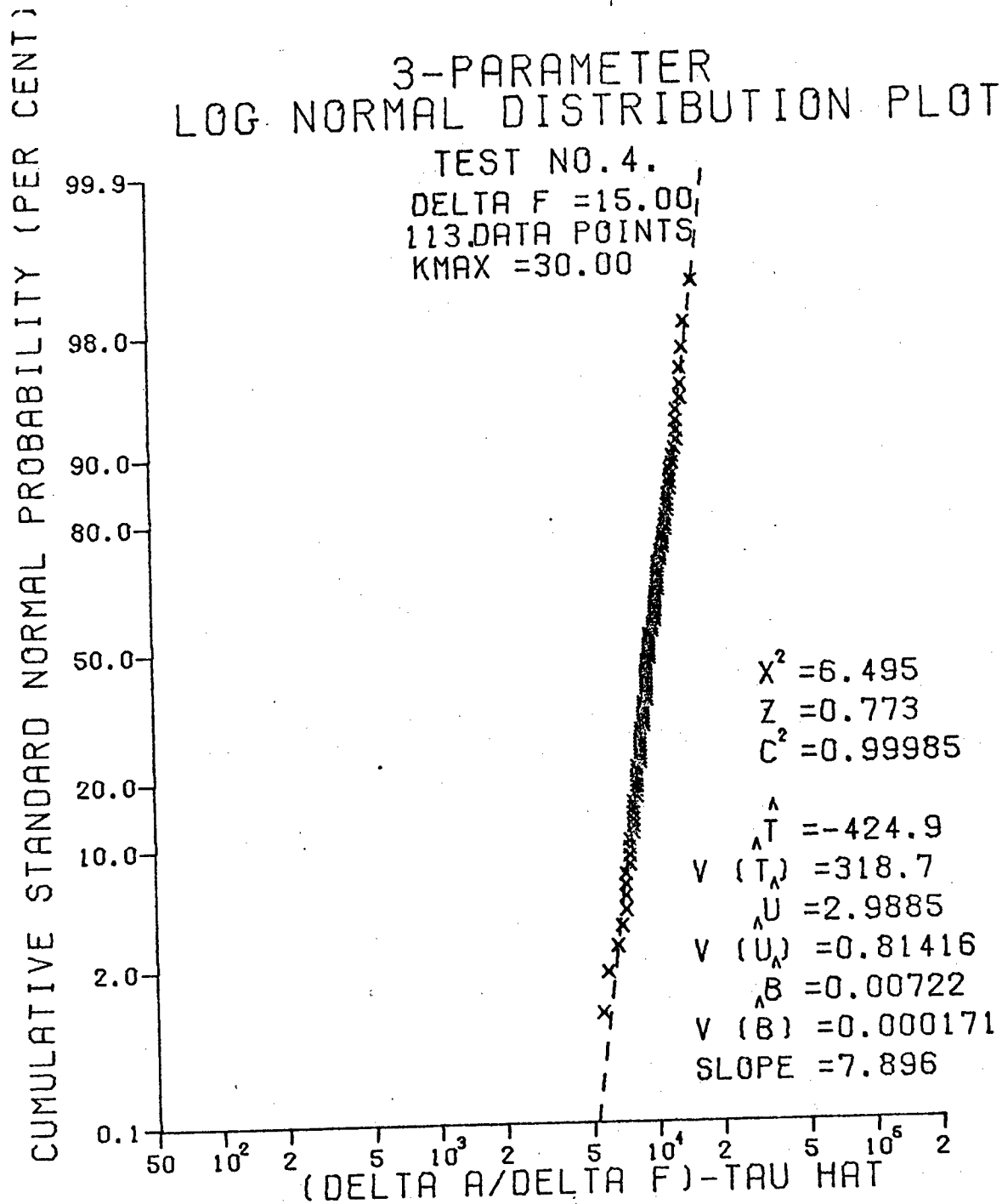


FIGURE 6. 3-PARAMETER LOGNORMAL DISTRIBUTION  
PLOT OF  $\Delta A / \Delta F$  DATA

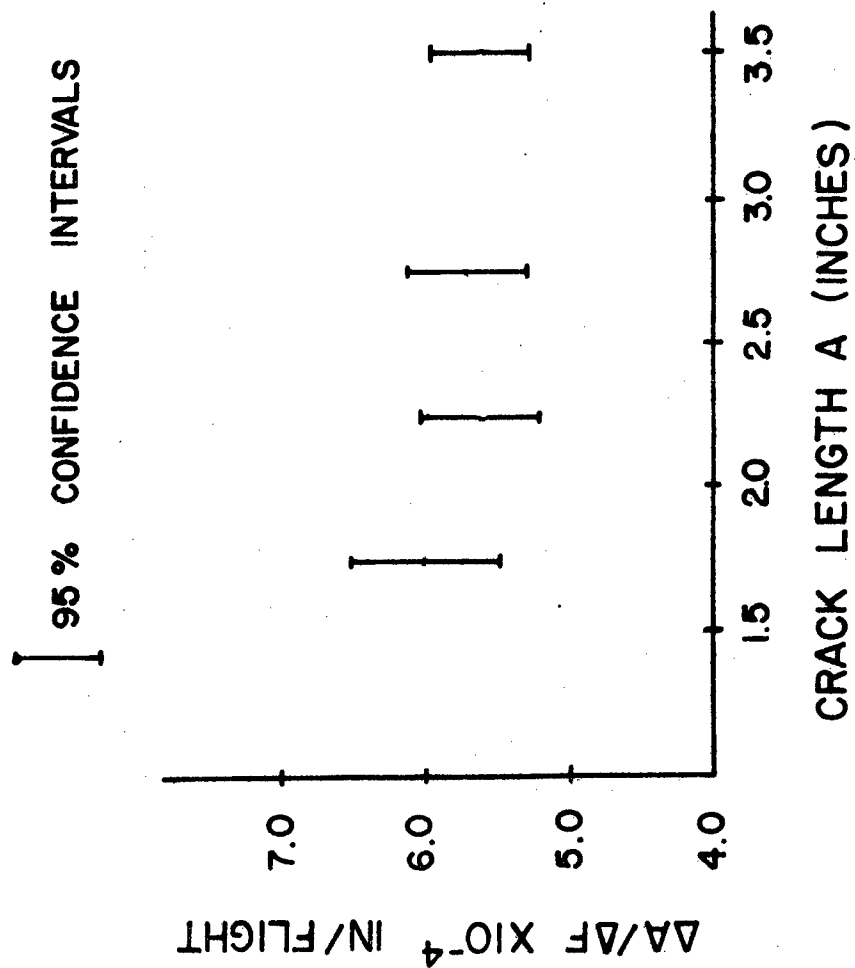


FIGURE 7: VARIATION IN CRACK GROWTH RATE AS A FUNCTION OF CRACK LENGTH

The largest set of contiguous points was used to evaluate the  $\Delta a/\Delta F$  distribution. The sample means and standard deviations are shown in Table II. Successive points were then removed to evaluate the effect of increasing the crack length increment on the mean and variance.

Three differentiation schemes were employed to investigate the variability in the  $\Delta a/\Delta F$  data. The sample means and standard deviations resulting from the secant and two seven point removable strip differentiation methods are shown in Table II. When "a" versus "F" data were fitted with a polynomial and then differentiated, the mean remained constant; the variance, however, was greatly reduced in all cases because of artificial smoothing induced through this process.

By removing selected points and differentiating using the secant method, the effect of taking larger crack growth length increment measurements was investigated. Through this process, the sample mean of  $\Delta a/\Delta F$  is expected to remain the same. Neulieb<sup>(9)</sup> showed that for constant stress intensity factor, steady state conditions, and assuming independence, the coefficient of variation ( $v$ ) should decrease as one over the square root of the number of increments which were added. In this study, the standard deviations as shown in Figure 8 decrease faster than would occur for independent crack growth intervals. It is, therefore, concluded that crack growth in a given increment may be dependent on what has occurred in a previous increment. An interval of fast crack growth always followed by an interval of slow crack growth or vice versa is an example of dependence. In the case of dependent data, the coefficient of variation will decrease at a faster rate than

TABLE II Sample Means and Variances of FCGR Data  
for Successively Removing FCG Data Points  
from Fig. 3 Data Set

FCGR DATA FOR METHOD OF DIFFERENTIATION									
FCG Points in Set	Effective Measurement Frequency ( $\Delta F$ ) Flights	Mean Crack Growth Increment ( $\Delta a$ ) inch	SECANT METHOD			INCREMENTAL POLYNOMIAL			
			Mean FCGR $10^{-4}$ inch /Flight	Standard Deviation $10^{-4}$ inch /Flight	Coefficient of Variation	Mean FCGR $10^{-4}$ inch /Flight	Standard Deviation $10^{-4}$ inch /Flight	Coefficient of Variation	
116	15	0.008	5.7	2.04	0.36	5.7	0.5	0.09	
58	30	0.017	5.7	1.44	0.25	5.6	0.42	0.08	
38	45	0.026	5.7	0.90	0.16	5.7	0.30	0.05	
29	60	0.034	5.6	0.66	0.12	5.6	0.35	0.06	
23	75	0.042	5.6	0.79	0.14	5.5	0.36	0.06	

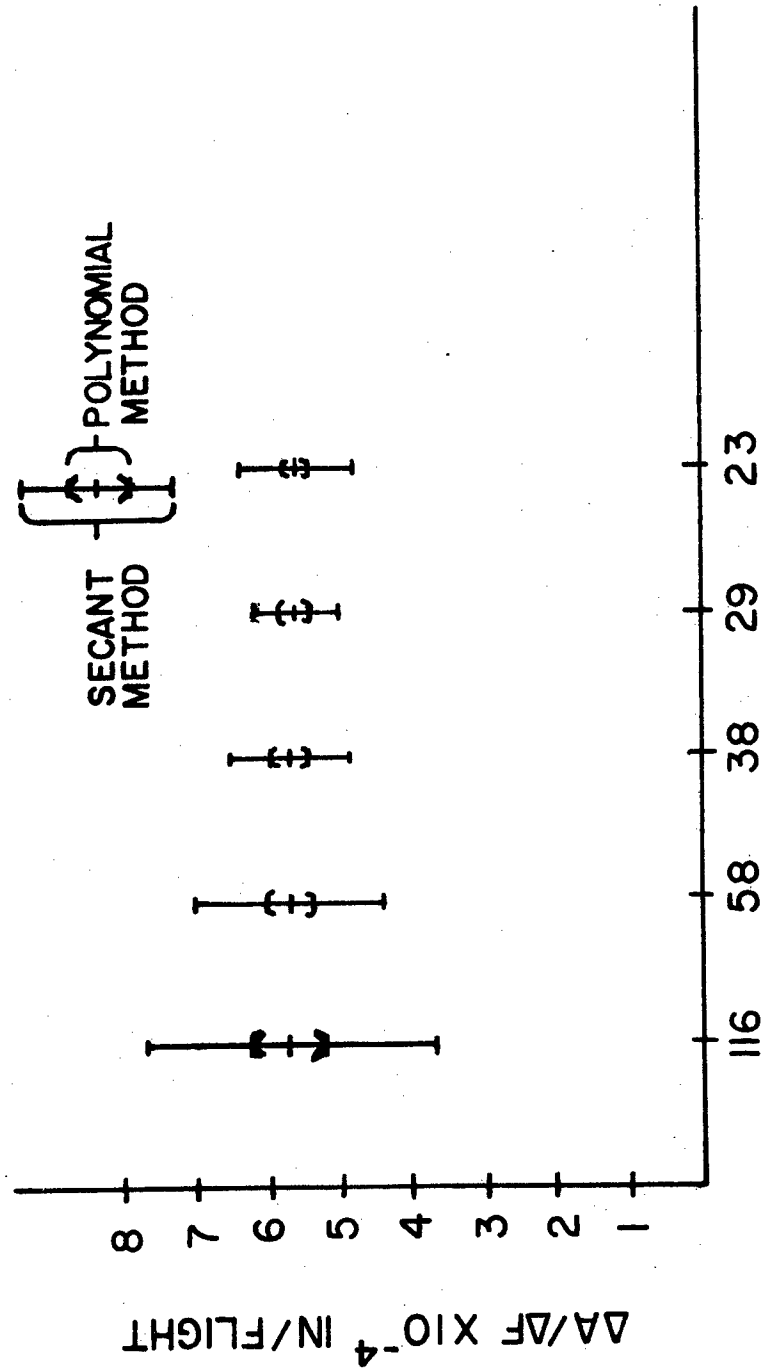


FIGURE 8: COMPARISON OF MEANS & STD. DEV. FOR SUCCESSIVELY REMOVED POINTS



than the square root of  $N$ . Dependency may be attributed to actual material behavior, measurement discrepancy or variability in load application.

The coefficients of variation shown in Table II reach an acceptable level of under twenty percent when the ratio of the crack length measurement increment,  $\Delta a$ , to the measurement precision,  $\epsilon$ , is around ten to one. A  $\Delta a/\epsilon$  ratio of 10/1 should be considered when making a determination of crack growth rates. The polynomial methods of differentiation did not aid in the assessment of variability in crack growth rate behavior.

Determination of the factors effecting the variance and their relative influence would require further study. It was discovered that a different value for the variance was found depending on which point one starts with in the differentiation scheme (Table III). This leads to a suggestion that the data may be periodic. This periodic variability is another indication of the possible dependency of the data.

# of increments added together	Starting Point in Set				
	1	2	3	4	5
1	$4.17(10)^{-8} \text{ in./Flight}$				
2	2.08	1.40			
3	0.813	1.32	0.76		
4	0.431	0.60	0.89	0.47	
5	0.624	0.36	0.38	0.57	0.29

TABLE III Variances Depending on Starting Point  
in the  $\Delta a/\Delta F$  Set.

## SECTION 4

### CONCLUSIONS

It was found that the flight-by-flight load history used for this study exhibited steady-state behavior. This was determined by the smoothness of the  $a$  vs.  $F$  data curve and the repeatability of the sample mean and standard deviation of the  $\Delta a/\Delta F$  data as the crack length increased. Because of this, a distribution function can be found which describes the  $\Delta a/\Delta F$  data.

In this flight-by-flight study, the three-parameter lognormal distribution function with a correlation coefficient of 0.999 best described the  $\Delta a/\Delta F$  data. The two-parameter lognormal distribution function with a correlation coefficient of 0.998 was also acceptable as was the normal distribution function with a correlation coefficient of 0.991. The normal distribution function, however, allows for negative crack growth which is physically unrealistic.

The method of differentiation had no effect on the mean crack growth rate. The variance, however, was affected by the method of differentiation. Using a polynomial method had the effect of decreasing the variance in comparison to the variance attained using the secant method of differentiation. This is to be expected because the polynomial method is a smoothing process. The secant method appears to be a more realistic choice in establishing crack growth rates.

Increasing the crack length measurement increment size had no significant effect on the sample mean. The variance decreased faster than would be expected for independent data. Something in the crack

growth testing process gives rise to dependency in the data. Possible contributors are the measurement technique, load shedding process and actual material behavior.

The coefficient of variation is related to the relative size of the measurement precision. A ratio of crack increment size to precision ( $\Delta a/\Delta F$ ) of ten to one gave an acceptable coefficient of variation of less than twenty percent. A smaller increment size gave a higher coefficient of variation. Larger increment sizes did not improve the variability. Increment measurement size can be selected to minimize the variability in flight-by-flight crack growth rate testing by ensuring that the crack length measurement interval is large compared to the measurement precision.

## SECTION 5

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